Anderson cross-localization in photonic lattices

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Abstract: We report two-dimensional Anderson localization in waveguide arrays with effectively one-dimensional disorder. Although the disorder is highly anisotropic it induces strong localization along both array axes, in which the waveguide spacing is regular or disordered.

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Particle dynamics in a disordered infinite medium is considered a key issue in solid-state physics. It was P. W. Anderson who showed that the scattering is a coherent phenomenon so that quantum-mechanical corrections have to be applied, to the possible extend of a complete suppression of diffusion also known as Anderson localization [1]. In particular optical setups proved to be promising candidates to observe coherent scattering in random media [2, 3, 4, 5, 6]. A breakthrough was the optical analogy of a solid and a photonic waveguide array (or lattice) due to an analogy between the Schrödinger equation and the paraxial Helmholtz equation: a longitudinally invariant, i.e. time-independent, disorder could be realized, as required for true Anderson localization [7, 8, 9].

Importantly, there are two different kinds of disorder, that can be introduced in an optical waveguide lattice. One way is to alter randomly the refractive index of each guide, that corresponds to changing on-site energy of the individual potential wells in quantum mechanics, yielding so called *on-diagonal* disorder. The other possibility is the random fluctuation of the inter-site spacing between identical lattice sites, resulting in *off-diagonal* disorder (ODD). It is commonly accepted that, in the latter case, Anderson localization only occurs in the direction of the ODD.

In this work, we demonstrate, theoretically and experimentally, Anderson cross-localization (ACL), where in a two-dimensional (2D) setting an effective one-dimensional (1D) ODD is sufficient to localize the wave function in both transverse directions. We find that, although the disorder is strongly anisotropic and effectively 1D it couples both transverse dimensions in the optical lattice and induces strong Anderson localization along both array axes, in which the waveguide spacing is either regular or disordered, and the degree of localization in the direction without disorder remains only slightly weaker than in the direction in which disorder is acting.

In order to describe ACL in disordered 2D waveguide arrays we employ the nonlinear Schrödinger equation for the dimensionless light amplitude q:

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\left(\frac{\partial^2 q}{\partial \eta^2} + \frac{\partial^2 q}{\partial \zeta^2}\right) - q|q|^2 - pR(\eta,\zeta), \tag{1}$$

where η, ζ are the transverse coordinates normalized to the characteristic beam width, ξ is the propagation distance normalized to the diffraction length and the parameter p describes the refractive index contrast. The function $R(\eta,\zeta) = \sum_{k,m} \exp\left[-\frac{(\eta-\eta_k)^4}{w_\eta^4} - \frac{(\zeta-\zeta_m)^4}{w_\zeta^4}\right]$ describes the refractive index distribution in the disordered array, with w_η, w_ζ are the widths of individual waveguides along the horizontal η and vertical ζ axes, respectively. We additionally assume that the disorder is introduced only to the position of the waveguide centers. This ODD affects only the coupling between the waveguides without modifying the propagation constant of the light in the individual guides. Thus, while $\zeta_m = md_\zeta$ where d_ζ is the regular waveguide spacing along ζ axis and $m \in \mathcal{N}$, the coordinates of the waveguides along the η axis are given by $\eta_k = d_\eta + r_k$, where d_η is the regular waveguide spacing, $k \in \mathcal{N}$, and $r_k < d_\eta/2$ is a random shift of the k-th waveguide center, that is uniformly distributed in $[-S_\eta, +S_\eta]$. Hence, the degree of disorder in such a system is controlled by the parameter S_η determining the maximal possible shift of the waveguides in all realizations of an array from the considered ensemble.

The output intensity distributions obtained numerically by averaging over the ensemble of $n = 10^3$ arrays and experimentally by averaging the measured near-field distribution of 25 realizations are shown in Fig. 1 a) - c).

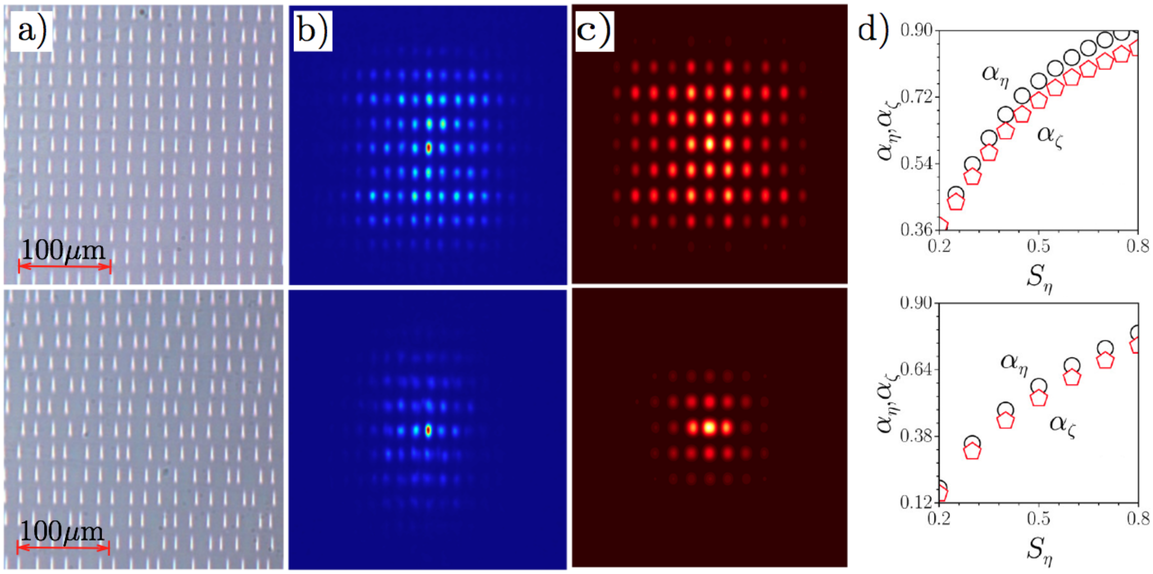

Fig. 1. a)-c): Microscopic images of disordered waveguide arrays, experimental and theoretical averaged output intensity distributions in linear case. The disorder level is $S_\eta = 0.2$ and 0.4 for upper and lower row respectively. d): Theoretically calculated (upper) and experimentally determined (lower) exponential decay rates α_η and α_ζ of the averaged output intensity distributions versus disorder strength S_η.

The central result of our work is that increasing the disorder level surprisingly causes simultaneous and almost equally strong Anderson localization along both η and ζ axes, as one observes from numerical simulations from Fig. 1 c), despite the fact that disorder is strongly anisotropic and effectively 1D (only the η coordinate of the waveguides fluctuate). Although such a "cross-localization" effect arises because introducing a 1D disorder in the separation between the individual sites (waveguides) in any multidimensional periodic system inevitably modifies also the site-to-site separation also in all orthogonal directions, affecting the coupling also in these directions, the almost equal degree of localization in the vertical and horizontal directions is particularly surprising taking into account the fact that at relatively small disorder levels $S_\eta = 0.1$ the variation of the horizontal waveguide separation S_η causes much smaller variation of the distance between waveguides in neighboring rows $S_\eta^2/2d_\zeta$, which intuitively should results in much weaker localization along the ζ axis. This surprising behavior was proven in our experiments. We fabricated arrays with 21×21 waveguides for several disorder levels S_η (Fig. 1 a)) employing the laser direct-writing technology [10]. The excellent agreement to our numerical simulation is shown in the experimental results Fig. 1 b). Although the degree of localization in both direction is almost equally, in the direction without disorder it always remains slightly weaker than that in the direction in which disorder is introduced as one can see from the exponential decay rates in Fig. 1 d).

In conclusion we demonstrated, Anderson cross-localization), where in a 2D setting an effectively 1D ODD is sufficient to localize the wave function in both transverse directions. We find that, although the disorder is highly anisotropic it induces strong Anderson localization along both array axis, in which the waveguide spacing is either regular or disordered.

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